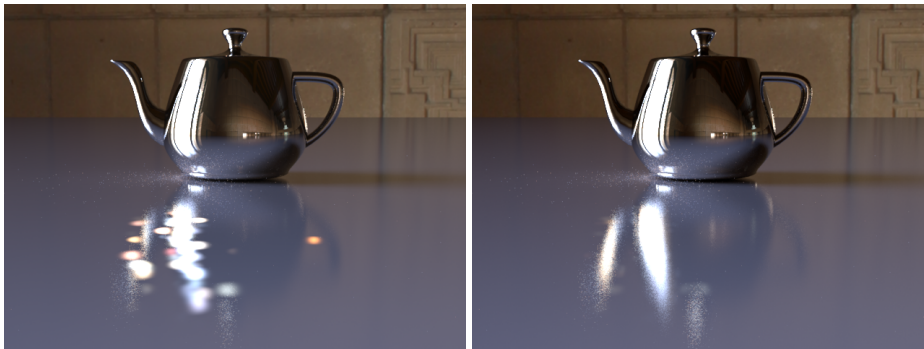# Photon Density Estimation using Multiple Importance Sampling

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Figure 1: *A perfect specular teapot on a glossy surface. These images are rendered using path tracing and caustics photon map. Left: general density estimation. Right: our method. They are generated with the same number of photons (70000 photons) and the same sample rays (640×480 pixels, 8 oversamples/pixel, max 500 paths/pixel). The rendering times of the left and the right are 218 seconds and 225 seconds, respectively (CPU:Core2 Duo 3GHz). Our method produces less noise than the general method in spite of lower overheads.*

1 Introduction

In global illumination rendering, final gathering (or path tracing) and caustics photon map [Jensen 2001] are frequently used. However, photon mapping generate estimation results with many noises on glossy surfaces as in the left image of Figure 1. We solve this problem using MIS (multiple importance sampling) [Veach 1997]. The advantages of our method are an easy implementation, lower overheads, and good estimation results without delicate parameter tuning.

MIS is often used to calculate direct illumination of area light sources. However, our target is caustics photons emitted from area light sources or environment maps. In this paper, we introduce a weighting function of density estimation for MIS.

2 Multiple Importance Sampling

MIS is a strategy to combine multi-sample models using a weighting function. Let w_i is the weighting function of sample model i:

$$w_i = \frac{q_i{}^\beta}{\sum_k q_k{}^\beta} \qquad (1)$$

This is called power heuristic. q_i is the product of the sample number and the PDF (probability distribution function) of sample model i. β is the exponent parameter of this heuristic. In this paper, we use $\beta = 2$ because [Veach 1997] says it is reasonable value.

3 Weight of Photon Density Estimation

In calculation of caustics, we combine density estimation and sample rays using MIS. Let w_1 be the weight of density estimation, and let w_2 be the weight of sample rays distributed according to the BRDF. In this paper, we introduce the following q_1 (See Appendix)

$$q_1 = \frac{N L_e(x', \vec{\omega}') \cos\theta \Delta A}{\Phi \alpha} \qquad (2)$$

where N is the number of emitted photons, and the pair of position x' and direction $\vec{\omega}'$ is the sample point on the light source, and $L_e(x', \vec{\omega}')$ is the emitted radiance of this sample point, and θ is the angle between the incident direction and the surface normal, and ΔA is the area covered by the nearest photons, and Φ is the total radiant flux of light sources, and α is a parameter for oversampling. When a ray first intersects a non-perfect specular surface, α is the same as number of oversamples per pixel, otherwise $\alpha = 1$. Most of nearest photons are the same for every sampling point in one pixel. Therefore, by dividing the density by α, we obtain the appropriate weighting function regardless of number of oversamples.

To calculate this weighting function, we store $L_e(x', \vec{\omega}')$ in each photon. Therefore photon data structure increase by 1 float size.

4 Results

In Figure 1, the material of the teapot is perfect specular reflection model, and the material of the floor is diffuse and glossy model. The caustics of the left image and the right image are rendered by the general density estimation and our method, respectively. The general density estimation produces spots on the glossy floor. On the other hand, our method reduces these spots because weights of photons that generate them are lower on the glossy floor.

5 Conclusion

We applied MIS to photon density estimation for rendering caustics. To calculate the weighting function of this MIS, we stored emitted radiance of the light source in each photon. As a result, noise of photon mapping could be reduced in spite of lower overheads. Especially, our method is effective on glossy surfaces. Also, we believe it is useful for scenes that contain LSDSE paths.

References

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VEACH, E. 1997. *Robust Monte Carlo Methods for Light Transport Simulation*. PhD thesis, Stanford University.

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