Unbiased VNDF Sampling for Backfacing Shading Normals

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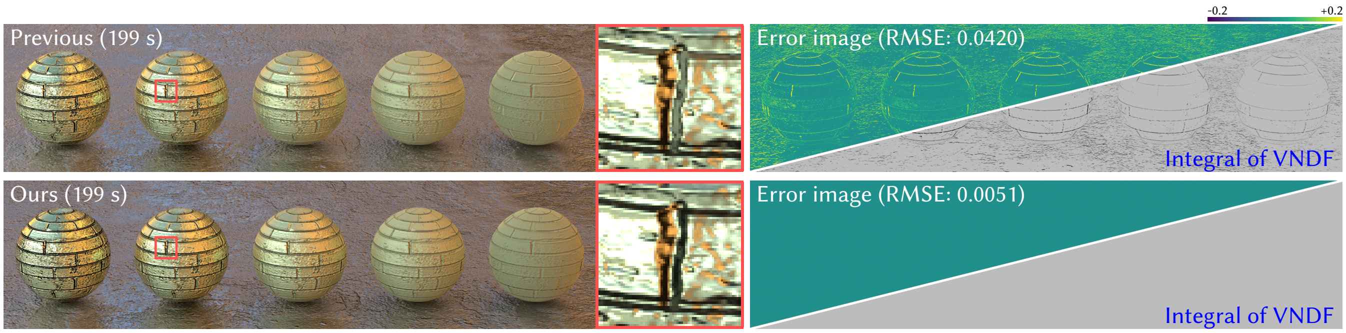

Figure 1: Path tracing using Smith–GGX VNDF sampling [Heitz 2018] with previous normalization (upper row) and our normalization (lower row) for a normal-mapped scene (2048 spp, CPU: AMD Ryzen™ 7 3800X). Right images are visualizations of the error and the VNDF integral. For backfacing shading normals, previous normalization produces a brightening bias. This is because the VNDF integral is less than one in these cases. We avoid this bias by correcting the normalization factor.

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1 INTRODUCTION

Shading normals are widely used in computer graphics productions (e.g., movies and video games) to improve the visual appearance without increasing the complexity of geometry. However, the difference between shading normals and geometry normals can violate physically based assumptions used in rendering techniques. One problem caused by this violation is that a bias can be produced in importance sampling according to a visible normal distribution function (VNDF) [Heitz and d'Eon 2014] when a shading normal is backfacing to an incident direction. This is because a *masking function* is directly used as the normalization factor of the VNDF, while most masking functions are modeled assuming a geometry normal.

In this talk, we present a corrected normalization factor to avoid the bias in VNDF sampling for the Smith [1967] and V-cavity [Cook and Torrance 1982] masking models. Our key insight is that in these models microfacets can be assumed to be *single sided* and partially visible from below the horizon (Fig. 2). Using our normalization factor derived from this single-sided assumption, we are able to perform unbiased VNDF sampling for backfacing shading normals. Our correction is easy to implement and has a negligible impact on performance especially for the Smith model.

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2 CLASSIC NORMALIZATION FOR THE VNDF

A VNDF $p(\mathbf{m};\mathbf{i})$ is a distribution of visible microfacet normals $\mathbf{m}$ observed from an incident direction $\mathbf{i}$, and it is given by

$$p(\mathbf{m};\mathbf{i}) = \frac{G(\mathbf{i},\mathbf{m})D(\mathbf{m})\max(\mathbf{i}\cdot\mathbf{m},0)}{|\mathbf{i}\cdot\mathbf{n}|}, \quad (1)$$

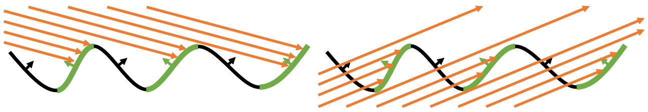
where $\mathbf{n}$ is the shading normal, $D(\mathbf{m}) \in [0,\infty)$ is the NDF, and $G(\mathbf{i},\mathbf{m}) \in [0,1]$ is the masking function that acts as a normalization factor. Since the VNDF is a probability distribution function (PDF) for VNDF sampling, its integral must be 1 as follows:

$$\int_{\mathcal{S}^2} p(\mathbf{m};\mathbf{i})\mathrm{d}\mathbf{m} = 1. \quad (2)$$

One famous masking model to satisfy this constraint for frontfacing shading normals is the following Smith model:

$$G(\mathbf{i},\mathbf{m}) = \frac{1}{1+\Lambda(\mathbf{i})}\chi^{+}(\mathbf{i}\cdot\mathbf{m}),$$

where $\chi^{+}(\mathbf{i}\cdot\mathbf{m})$ is the Heaviside function: 1 if $\mathbf{i}\cdot\mathbf{m} > 0$, 0 if $\mathbf{i}\cdot\mathbf{m} \leq 0$. For the GGX NDF, previous work [Heitz 2014; Walter et al. 2007] derived $\Lambda(\mathbf{i}) = -0.5 + 0.5\sqrt{\alpha_x^2 i_x^2/i_z^2 + \alpha_y^2 i_y^2/i_z^2 + 1}$ by assuming a frontfacing geometry normal (i.e., $i_z > 0$), where $[i_x, i_y, i_z]$ is the incident direction in tangent space, and $[\alpha_x, \alpha_y] \in (0,\infty)^2$ are roughness parameters. However, this closed-form masking function does not satisfy Eq. (2) for backfacing shading normals. An existing closed-form Smith–Beckmann model and the V-cavity model also have the same problem (please see the supplementary material).



(a) View from above the horizon (b) View from below the horizon

Figure 2: Single-sided microsurface. Incident rays (orange) pass through backfacing microfacets (black) and hits only frontfacing microfacets (green).

3 SINGLE-SIDED MICROSURFACE MODEL

For backfacing shading normals, we assume that microfacets are single-sided and partially visible from below the horizon. Thus, a ray may not hit a microfacet when a shading normal is backfacing to the incident direction of the ray (Fig. 2b). Since visible normals are microfacet normals hit by rays, the VNDF for this case is a conditional PDF when a ray hits a microfacet as follows:

$$\boxed{p(\mathbf{m};\mathbf{i}) = \frac{G(\mathbf{i},\mathbf{m})D(\mathbf{m})\max(\mathbf{i}\cdot\mathbf{m},0)}{H(\mathbf{i})|\mathbf{i}\cdot\mathbf{n}|}},$$

where $H(\mathbf{i}) \in [0,1]$ is the probability of ray hit. This hit probability is given by the ratio of the visible microsurface area to the macrosurface area projected onto the incident direction, as follows:

$$H(\mathbf{i}) = \frac{\int_{\mathcal{S}^2} G(\mathbf{i},\omega)D(\omega)\max(\mathbf{i}\cdot\omega,0)\mathrm{d}\omega}{|\mathbf{i}\cdot\mathbf{n}|}.$$

Unlike Eq. (1), our normalization factor is $\frac{G(\mathbf{i},\mathbf{m})}{H(\mathbf{i})}$. For frontfacing shading normals, a ray always hits a microfacet (i.e., $H(\mathbf{i}) = 1$), therefore this normalization factor is equivalent to $G(\mathbf{i},\mathbf{m})$ in this case. On the other hand, for backfacing shading normals, the normalization factor can be greater than the masking function due to the hit probability $H(\mathbf{i}) \leq 1$ while satisfying the constraint of the VNDF (Eq. 2).

3.1 Normalization Factor

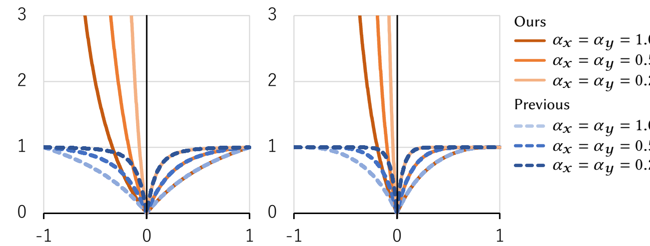
For the Smith model, $\Lambda(\mathbf{i})$ was derived assuming a frontfacing geometry normal. Therefore, the true $\Lambda(\mathbf{i})$ is not obvious for backfacing shading normals. Even for this case, we yield the normalization factor without deriving $\Lambda(\mathbf{i})$ as follows:

$$\frac{G(\mathbf{i},\mathbf{m})}{H(\mathbf{i})} = \frac{|\mathbf{i}\cdot\mathbf{n}|\frac{1}{1+\Lambda(\mathbf{i})}\chi^+(\mathbf{i}\cdot\mathbf{m})}{\int_{\mathcal{S}^2}\frac{1}{1+\Lambda(\mathbf{i})}\chi^+(\mathbf{i}\cdot\omega)\,D(\omega)\max(\mathbf{i}\cdot\omega,0)\mathrm{d}\omega}$$
$$= \frac{|\mathbf{i}\cdot\mathbf{n}|}{\int_{\mathcal{S}^2}D(\omega)\max(\mathbf{i}\cdot\omega,0)\mathrm{d}\omega}\chi^+(\mathbf{i}\cdot\mathbf{m})\,.$$

The right side is equivalent to the Smith masking function for frontfacing shading normals [Heitz 2014]. We derive the closed-form solution of this normalization factor for the GGX NDF for arbitrary shading normals as follows:

$$\boxed{\frac{G(\mathbf{i},\mathbf{m})}{H(\mathbf{i})} = \frac{2|i_z|}{i_z + \sqrt{\alpha_x^2 i_x^2 + \alpha_y^2 i_y^2 + i_z^2}}\chi^+(\mathbf{i}\cdot\mathbf{m})}.$$

For the case in the Beckmann NDF, please see the supplementary material. As shown in Fig. 3, existing closed-form masking functions



(a) Smith–GGX (b) Smith–Beckmann

Figure 3: Plots of normalization factors. The horizontal axis is i_z. For $i_z \geq 0$, our normalization factor (solid line) is identical to the existing closed-form masking function (dashed line). On the other hand, for $i_z < 0$, ours is larger than the masking function and approaches to infinity for $i_z \to -1$.

and our normalization factors are identical for $i_z \geq 0$, while they are significantly different for $i_z < 0$ (i.e., backfacing shading normal).

For the V-cavity model, the masking function is available for backfacing shading normals. Thus, we obtain the normalization factor for the V-cavity model in a straightforward way. A V-cavity VNDF sampling routine taking this normalization into account is shown in the supplementary material.

4 RESULTS

Fig. 1 shows rendering results using VNDF sampling for the Smith–GGX BRDF. Experimental results for the Smith–Beckmann and V-cavity BRDFs are shown in the supplementary material. For backfacing shading normals, the previous normalization produces a brightening bias due to the VNDF integral being less than one. Our normalization avoids this bias without noticeable performance degradation. For the Smith model, we correct only the normalization factor and do not need to modify existing VNDF sampling routines [Heitz 2018; Jakob 2014]. Therefore, our correction is easy to implement for the Smith model.

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