Stochastic Light Culling for Single Scattering in Participating Media: Supplemental Document

Shin Fujieda Yusuke Tokuyoshi Takahiro Harada

Advanced Micro Devices, Inc.

1. False Positives for Point Light Experiment

Since tile-based light culling produces false positives, we visualize them in Fig. 1 for clarification. The scene is the same as Fig. 1b in the main document and rendered with the same settings (i.e., one second rendering time with 380 samples per pixel). The heat color from blue to red represents the false discovery rate of 0% to 100%.

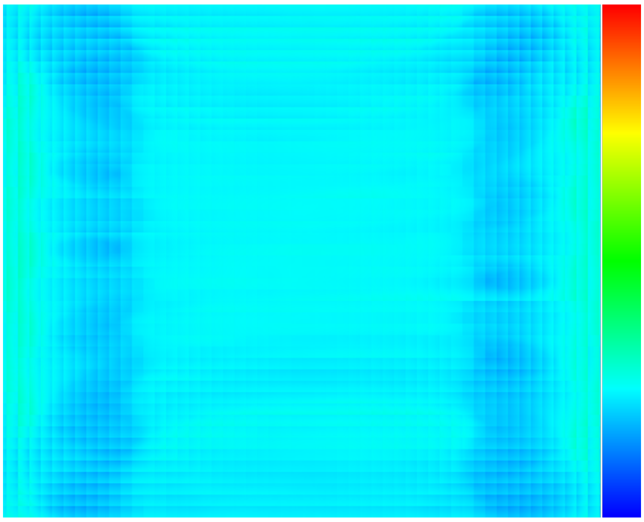

Figure 1: *False discovery rate for our method with tile-based light culling on a 2,560 point light scene (same scene as Fig. 1b in the main document).*

2. Acceptance Probability for Diffuse VPLs

2.1. Distance Sampling

In this paper, we can use different distance sampling techniques for the inside and outside of the inner sphere. Therefore, within the inner sphere, we use equiangular sampling ignoring the directionality of radiant intensity for simplicity. Improvement in the distance sampling inside the inner sphere is our future work.

For the outside the inner sphere, we perform Russian roulette according to the acceptance probability $P(\mathbf{d}, l)$ where $\mathbf{d} \in S^2$ and $l \in [0, \infty)$ are the direction and distance from the VPL (virtual point light), respectively. This Russian roulette is done by the intersection test between the ray and the isosurface of emissive radiance. If this isosurface is not a sphere, a complex intersection test is required after light culling. To simplify the intersection test, we use an acceptance probability according to a directional term given by the bound of the isosurface as follows:

$$P(\mathbf{d}, l) = \min\left(\frac{I_{\max}(\mathbf{d})}{\alpha l^2}, 1\right), \tag{1}$$

where $I_{\max}(\mathbf{d})$ is the directional term that bounds the original radiant intensity $I(\mathbf{d}) = \frac{\Phi}{\pi} \max(\mathbf{d} \cdot \mathbf{n}, 0)$, and the isosurface of this probability is equal to the bounding sphere whose center and radius are given by

$$\mathbf{c}_o = \mathbf{x} + \left(\frac{1}{3}\right)^{\frac{3}{4}} \sqrt{\frac{\Phi}{\pi\alpha\xi}}\mathbf{n}, \tag{2}$$

$$r_o^{\text{area}} = \left(\frac{4}{27}\right)^{\frac{1}{4}} \sqrt{\frac{\Phi}{\pi\alpha\xi}}. \tag{3}$$

For notations, please see the main document. With this sphere, the directional term $I_{\max}(\mathbf{d})$ can be obtained (see Sec. 2.2 for derivation), but our method does not require the explicit form of $I_{\max}(\mathbf{d})$ in practice. This is because we use a distance sampling according to $I_{\max}(\mathbf{d})/(l^2 P(\mathbf{d}, l))$ which results in uniform sampling. Thus, our method does not require the explicit form of $I_{\max}(\mathbf{d})$ and complex distance sampling routines according to $I_{\max}(\mathbf{d})$ for outside the inner sphere.

To select segments inside or outside the inner sphere, we use the same weight as Eqs. 6–8 in the main document for simplicity. Since this selection ignores the directionality of the radiant intensity, it can produce an additional variance. Variance reduction for this segment selection is also our future work.

2.2. Derivation of the Directional Term

Although our method does not require the explicit form of the radiant intensity bound $I_{\max}(\mathbf{d})$, this section derives $I_{\max}(\mathbf{d})$ for diffuse VPLs for ease of understanding. Since the acceptance probability (Eq. 1) is inversely proportional to the squared distance, $I_{\max}(\mathbf{d})$

should be proportional to the squared distance from the VPL position to the bounding sphere (Eqs. 2 and 3). We derive this distance using the sphere-line intersection. Let us transform the bounding sphere to a unit sphere whose center is the origin, and then the VPL position is transformed to $\mathbf{x} = -\frac{1}{\sqrt{2}}\mathbf{n}$. Therefore, the point $\mathbf{p}$ on the transformed line is given by

$$\mathbf{p} = \tau\mathbf{d} - \frac{1}{\sqrt{2}}\mathbf{n}, \tag{4}$$

where τ is the distance from the VPL in the transformed space. The intersection point of this line and the unit sphere is given as $\|\mathbf{p}\|^2 = 1$. It is rewritten into a quadratic equation:

$$\tau^2 - \sqrt{2}(\mathbf{d}\cdot\mathbf{n})\tau - \frac{1}{2} = 0. \tag{5}$$

The positive solution of this equation is given by

$$\tau = \frac{(\mathbf{d}\cdot\mathbf{n}) + \sqrt{(\mathbf{d}\cdot\mathbf{n})^2+1}}{\sqrt{2}}. \tag{6}$$

Since the original radiant intensity is $I(\mathbf{d}) = \frac{\Phi}{\pi}\max(\mathbf{d}\cdot\mathbf{n},0)$ and the bounding radius for $\max(\mathbf{d}\cdot\mathbf{n},0)$ is $\left(\frac{4}{27}\right)^{\frac{1}{4}}$ [DS06], we yield the bound of the radiant intensity:

$$I_{\max}(\mathbf{d}) = \frac{\Phi}{\pi}\left(\left(\frac{4}{27}\right)^{\frac{1}{4}}\tau\right)^2 = \frac{\Phi\left((\mathbf{d}\cdot\mathbf{n}) + \sqrt{(\mathbf{d}\cdot\mathbf{n})^2+1}\right)^2}{3\sqrt{3}\pi}. \tag{7}$$

References

[DS06] Dachsbacher, C. and Stamminger, M. "Splatting Indirect Illumination". I3D '06. 2006, 93–100 2.